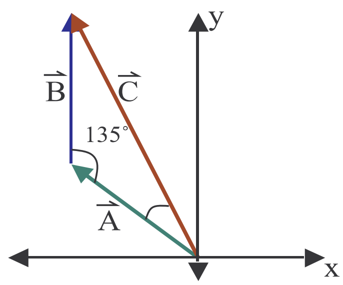
## Chapter 3 Problem 13 †




**Given**
$\vec{A} = 360\ km\ \angle 135°$
$\vec{B} = 400\ km\ \angle 90°$

**Solution**
Find the displacement from the start.

Solving this graphically you get the diagram given above. We can now use the law of cosines to find the magnitude of vector C.

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

$$C^2 = (360\ km)^2 + (400\ km)^2 - 2(360\ km)(400\ km)\cos 135°$$
$$C^2 = 493,247\ km^2$$
$$C = 702\ km$$

Next find the angle *beta* using the law of sines.

$$\frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

$$\beta = \sin^{-1}\left(\frac{B\sin\gamma}{C}\right)$$

$$\beta = \sin^{-1}\left(\frac{400\ km \sin 135°}{702\ km}\right) = 23.8°$$

Since the direction of $\vec{C}$ is and angle $\beta$ less than the angle of $\vec{A}$, the direction of $C$ is

$$\angle C = 111.2°$$

The problem can also be solved using unit vectors.
First convert into unit vector notation.

$$\vec{A} = \left\{A\cos\theta\hat{\imath} + A\sin\theta\hat{\jmath}\right\} = \left\{360\cos 135\hat{\imath} + 360\sin 135\hat{\jmath}\right\} km$$

$$\vec{A} = \left\{-254.6\hat{\imath} + 254.6\hat{\jmath}\right\}\ km$$

and

$$\vec{B} = \left\{B\cos\theta\hat{\imath} + B\sin\theta\hat{\jmath}\right\} = \left\{400\cos 90\hat{\imath} + 400\sin 90\hat{\jmath}\right\}\ km$$

†Problem from Essential University Physics, Wolfson

$$\vec{B} = \left\{400\hat{\jmath}\right\}\ km$$

Add the two vectors together to get the displacement.

$$\vec{C} = \vec{A} + \vec{B} = \left\{-254.6\hat{\imath} + 254.6\hat{\jmath}\right\} km + \left\{400\hat{\jmath}\right\}\ km$$

$$\vec{C} = \left\{-254.6\hat{\imath} + 654.6\hat{\jmath}\right\}\ km$$

Convert into polar coordinate notation.

$$C = \sqrt{(-254.6\ km)^2 + (654.6\ km)^2}$$

$$C = 702\ km$$

$$\theta = \tan^{-1}\left(\frac{654.6\ km}{-254.6\ km}\right) = -68.7^\circ$$

However, our answer should be in the 2nd quadrant not the fourth; therefore, add 180° to give

$$\vec{C} = 702\ km\ \angle 111.3^\circ$$