

## Chapter 2 Problem 48 †

### Given

$$x = bt + ct^3$$

$$b = 1.50 \text{ m/s}$$

$$c = 0.640 \text{ m/s}^3$$

### Solution

a) Find the average velocity between 1.00 s and 3.00 s.

At  $t_i = 1.00 \text{ s}$ ,

$$x_i = (1.50 \text{ m/s})(1.00 \text{ s}) + (0.640 \text{ m/s}^3)(1.00 \text{ s})^3 = 2.14 \text{ m}$$

At  $t_f = 3.00 \text{ s}$ ,

$$x_f = (1.50 \text{ m/s})(3.00 \text{ s}) + (0.640 \text{ m/s}^3)(3.00 \text{ s})^3 = 21.78 \text{ m}$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{21.78 \text{ m} - 2.14 \text{ m}}{3.00 \text{ s} - 1.00 \text{ s}} = 9.82 \text{ m/s}$$

b) Find the average velocity between 1.50 s and 2.50 s.

At  $t_i = 1.50 \text{ s}$ ,

$$x_i = (1.50 \text{ m/s})(1.50 \text{ s}) + (0.640 \text{ m/s}^3)(1.50 \text{ s})^3 = 4.41 \text{ m}$$

At  $t_f = 2.50 \text{ s}$ ,

$$x_f = (1.50 \text{ m/s})(2.50 \text{ s}) + (0.640 \text{ m/s}^3)(2.50 \text{ s})^3 = 13.75 \text{ m}$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{13.75 \text{ m} - 4.41 \text{ m}}{2.50 \text{ s} - 1.50 \text{ s}} = 9.34 \text{ m/s}$$

c) Find the average velocity between 1.95 s and 2.05 s.

At  $t_i = 1.95 \text{ s}$ ,

$$x_i = (1.50 \text{ m/s})(1.95 \text{ s}) + (0.640 \text{ m/s}^3)(1.95 \text{ s})^3 = 7.671 \text{ m}$$

At  $t_f = 2.05 \text{ s}$ ,

$$x_f = (1.50 \text{ m/s})(2.05 \text{ s}) + (0.640 \text{ m/s}^3)(2.05 \text{ s})^3 = 8.589 \text{ m}$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{8.589 \text{ m} - 7.671 \text{ m}}{2.05 \text{ s} - 1.95 \text{ s}} = 9.18 \text{ m/s}$$

---

†Problem from Essential University Physics, Wolfson

d) Find the instantaneous velocity at  $t = 2.00 \text{ s}$ .

$$v = \frac{dx}{dt} = \frac{d(bt + ct^3)}{dt} = b + 3ct^2$$

At  $t = 2.00 \text{ s}$ ,

$$v = (1.50 \text{ m/s}) + 3(0.640 \text{ m/s}^3)(2.00 \text{ s})^2 = 9.18 \text{ m/s}$$

Notice that as the interval decreases, it approaches the instantaneous velocity.