Chapter 15 Problem 55 †

Given
$P = 140\ kPa = 1.4 \times 10^5\ Pa$

Solution
Find the height reached by the water emerging from the holes.

Inside the hole the water is at the given gauge pressure with effectively no velocity. From Bernoulli's equation we see a relationship with water outside of the sprinkler.

$$P_i + \frac{1}{2}\rho v_i^2 + \rho g h_i = P_o + \frac{1}{2}\rho v_o^2 + \rho g h_o$$

Applying the conditions given for inside the sprinkler we have

$$P_i + \rho g h_i = P_o + \frac{1}{2}\rho v_o^2 + \rho g h_o$$

For water outside the sprinkler the pressure is just the atmospheric pressure. If we are using gauge pressure, then this pressure is zero.

$$P_i + \rho g h_i = \frac{1}{2}\rho v_o^2 + \rho g h_o$$

At this point we can go two routes in solving the problem. Method 1 assumes that the height inside and outside the sprinkler are the same. We can then find the velocity given the gauge pressure inside the sprinkler. Once we have the velocity we can use either kinematic equations or conservation of energy to solve for the height.

Method 2 uses Bernoulli's equation for the whole problem. Since conservation of energy is built into Bernoulli's equation, we can assume the velocity at the highest point outside the sprinkler to be zero. The equation is then

$$P_i + \rho g h_i = \rho g h_o$$

Solving for the change in height gives

$$P_i = \rho g h_o - \rho g h_i = \rho g (h_o - h_i)$$

$$h_o - h_i = \frac{P_i}{\rho g}$$

$$\Delta h = h_o - h_i = \frac{(1.4 \times 10^5\ Pa)}{(1.0 \times 10^3\ kg/m^3)(9.8\ m/s^2)}$$

$$\Delta h = 14.3\ m$$

†Problem from Essential University Physics, Wolfson