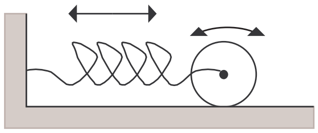
Chapter 13   Problem 63 †



Given
M - mass of wheel
R - radius of wheel
k - spring constant

Solution
Find the angular frequency of the rolling wheel attached to a spring.

For this problem we will assume the spring has negligible mass; therefore, we will not have to worry about the spring having any kinetic energy. From the concept of conservation of energy the total energy of the system will remain unchanged as long as there are no outside forces. The total energy of the system consist of the potential energy in the spring, the rotational kinetic energy of the wheel, and the translational kinetic energy of the wheel. Set the coordinate system such that $x = 0m$ at the equilibrium position of the spring. Assuming the wheel moves a small distance x to the right, the potential energy stored in the spring is

$$U = \tfrac{1}{2}kx^2$$

The velocity of the wheel will be dx/dt. The translational kinetic energy of the wheel is then

$$K_{tran} = \tfrac{1}{2}Mv^2 = \tfrac{1}{2}M\left(\frac{dx}{dt}\right)^2$$

The rotational kinetic energy is

$$K_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{1}{4}Mv^2 = \tfrac{1}{4}M\left(\frac{dx}{dt}\right)^2$$

The moment of inertia of the wheel is that of a solid disk and is obtained from Table 10-2. Combining these energies together gives

$$E_{total} = U + K_{tran} + K_{rot}$$

$$E_{total} = \tfrac{1}{2}kx^2 + \tfrac{1}{2}M\left(\frac{dx}{dt}\right)^2 + \tfrac{1}{4}M\left(\frac{dx}{dt}\right)^2$$

$$E_{total} = \tfrac{1}{2}kx^2 + \tfrac{3}{4}M\left(\frac{dx}{dt}\right)^2$$

Differentiating both sides with respect to time gives

$$\frac{dE_{total}}{dt} = \frac{d(\frac{1}{2}kx^2)}{dt} + \frac{d\left(\frac{3}{4}M\left(\frac{dx}{dt}\right)^2\right)}{dt}$$

$$0 = \tfrac{1}{2}k(2x)\frac{dx}{dt} + \tfrac{3}{4}M\left(2\frac{dx}{dt}\right)\frac{d^2x}{dt^2}$$

†Problem from Essential University Physics, Wolfson

$$0 = kx + \tfrac{3}{2}M\frac{d^2x}{dt^2}$$

$$M\frac{d^2x}{dt^2} = -\frac{2k}{3}x$$

This formula compares to 13-3 with a modified spring constant of $2k/3$. Substituting this into equation 13-7a gives an angular frequency of

$$\omega = \sqrt{\frac{2k}{3M}}$$