Chapter 13 Problem 47 [†]

Given
$F = 1.0\ pN$
$A = 15\ nm$

Solution
a) Find the spring constant of the mass-spring system.

As the molecule oscillates, its maximum force will correspond to the maximum displacement from equilibrium, which is the amplitude of the oscillation. Using Hooke's law gives

$$F = -kx$$

Solve for k gives (Ignore the negative sign. We are interested in the magnitudes not the direction of the force for this problem.)

$$k = \frac{F}{x}$$

At $x = A$ we have maximum displacement and, therefore, maximum force.

$$k = \frac{F_{max}}{A} = \frac{1.0 \times 10^{-12}\ N}{15 \times 10^{-9}\ m} = 6.67 \times 10^{-5}\ N/m$$

b) Find the effective mass of the mass-spring system.

The frequency is 70 Hz. The angular frequency is then

$$\omega = 2\pi f = 2\pi(70\ Hz) = 140\pi\ rad/s$$

Angular frequency of a mass-spring system is

$$\omega = \sqrt{\frac{k}{m}}$$

Solving for mass gives

$$m = \frac{k}{\omega^2}$$

Substituting in the provided values gives

$$m = \frac{6.67 \times 10^{-5}\ N/m}{(140\pi\ rad/s)^2} = 3.4 \times 10^{-10}\ kg$$

The effective mass is 0.34 micrograms.

[†]Problem from Essential University Physics, Wolfson