Chapter 13 Problem 24 †

Given

$f = 32,768\ Hz$

$A = 100nm = 1.0 \times 10^{-7}\ m$

Solution

Find the maximum velocity and acceleration of the face of the quartz crystal.

The position of the quartz crystal face is given by

$$x = A\cos(\omega t + \phi)$$

The maximum displacement is the amplitude, A. The angular frequency, ω, is caluclated from the frequency using the relationship $\omega_0 = 2\pi f$.

Taking the first derivative of the position function with respect to time gives the velocity,

$$v = -A\omega\sin(\omega t + \phi)$$

The maximum velocity is the amplitude times the angular frequency. Substituting in the appropriate values give a maximum velocity of

$$v_{\max} = A\omega = A2\pi f = (1.0 \times 10^{-7}\ m)2\pi(32,768\ Hz)$$

$$v_{\max} = 2.06 \times 10^{-2}\ m/s = 2.06\ cm/s$$

Taking the derivative of the velocity function gives the acceleration,

$$a = -A\omega^2\cos(\omega t + \phi)$$

The maximum acceleration is the amplitude times the angular frequency squared. Substituting in the appropriate values give a maximum acceleration of

$$a_{\max} = A\omega^2 = A\left(2\pi f\right)^2 = \left(1.0 \times 10^{-7}\ m\right)\left(2\pi(32,768\ Hz)\right)^2$$

$$a_{\max} = 4.24 \times 10^3\ m/s^2$$

†Problem from Essential University Physics, Wolfson