Chapter 12 Problem 24 [†]

Given
$h = 0.94x - 0.010x^2$

Solution
Find the equilibrium point of the roller coaster.

The equilibrium point corresponds with a minimum or maximum of the track. To find this critical point take the first derivative of the potential energy function and set it equal to zero. Then solve for x. The potential energy is

$$U = mgh = mg(0.94x - 0.010x^2)$$

$$\frac{dU}{dx} = \frac{d(mg(0.94x - 0.010x^2))}{dx} = 0$$

$$mg(0.94 - 0.020x) = 0$$

$$x = 47\ m$$

b) Determine the stability type of this equilibrium point.

Find the curvature of the function at the critical point. If it is upward curving (positive), then it is stable. If it is downward curving (negative), then it is unstable. If it is flat (zero), then it is neutrally stable.

$$\frac{d^2U}{dx^2} = \frac{d^2(mg(0.94x - 0.010x^2))}{dx^2}$$

$$\frac{d^2U}{dx^2} = -0.020mg$$

Since the mass and magnitude of the acceleration of gravity are positive, the second derivative is negative and the equilibrium point is unstable.

c) Find the height of the coaster at the equilibrium point.

Evaluate the height function at $x = 47\ m$.

$$h = 0.94(47\ m) - 0.010(47\ m)^2 = 22.1\ m$$

Notice that the units are not consistent. That is because there are implied units on the constants in the function for height.

[†]Problem from Essential University Physics, Wolfson