Chapter 11 Problem 49 †

Given

$m_l = 0.440\ kg$
$r_l = 0.035\ m$
$\omega_l = 180\ rpm$
$m_u = 0.270\ kg$
$r_u = 0.023\ m$
$\omega_u = 0\ rpm$

Solution

a) Find the final speed of the two disks when they come to a common speed.

Since we are working with angular momentum for this problem, it will not hurt to remain in rpm's for the angular velocity. The answer will, therefore, be in rpm's.
The moments of inertia for the disks are given by the relationship $I = \frac{1}{2}m \cdot r^2$. By conservation of angular momentum the initial angular momentum should equal the final angular momentum.

$$L_i = L_f$$

$$I_u\omega_u + I_l\omega_l = (I_u + I_l)\omega_f$$

Solving for the final angular momentum gives

$$\omega_f = \frac{I_u\omega_u + I_l\omega_l}{I_u + I_l} = \frac{\frac{1}{2}m_u r_u^2\omega_u + \frac{1}{2}m_l r_l^2\omega_l}{\frac{1}{2}m_u r_u^2 + \frac{1}{2}m_l r_l^2}$$

$$\omega_f = \frac{\frac{1}{2}(0.27\ kg)(0.023\ m)^2(0) + \frac{1}{2}(0.44\ kg)(0.035\ m)^2(180\ rpm)}{\frac{1}{2}(0.27\ kg)(0.023\ m)^2 + \frac{1}{2}(0.44\ kg)(0.035\ m)^2}$$

$$\omega_f = 142\ rpm$$

b) Find the fraction of initial kinetic energy lost.

$$loss = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = -\left(1 - \frac{K_f}{K_i}\right)$$

$$loss = -\left(1 - \frac{\frac{1}{2}(I_u + I_l)\omega_f^2}{\frac{1}{2}I_l\omega_l^2}\right) = -\left(1 - \frac{(I_u + I_l)\omega_f^2}{I_l\omega_l^2}\right)$$

$$loss = -\left(1 - \frac{(\frac{1}{2}m_u r_u^2 + \frac{1}{2}m_l r_l^2)\omega_f^2}{\frac{1}{2}m_l r_l^2\omega_l^2}\right) = -\left(1 - \frac{(m_u r_u^2 + m_l r_l^2)\omega_f^2}{m_l r_l^2\omega_l^2}\right)$$

$$loss = -\left(1 - \frac{((0.27\ kg)(0.023\ m)^2 + (0.44\ kg)(0.035\ m)^2)(143\ rpm)^2}{(0.44\ kg)(0.035\ m)^2(180\ rpm)^2}\right)$$

$$loss = -0.202$$

$$\%\ loss = -20.2\ \%$$

†Problem from Essential University Physics, Wolfson