Chapter 11 Problem 30[†]

$$\label{eq:generalized_states} \begin{split} & \mathbf{Given} \\ & |B| = 2|A| \\ & |\vec{A} \times \vec{B}| = A^2 \end{split}$$

Solution

Find the direction of vector B.

Since the cross product points in the negative z direction, the vector B must lie in the xy plane along with vector A. A negative z-direction corresponds to a clockwise rotation. Therefore, vector B must be pointed to the right of the direction of vector A as illustrated in the diagram. Finally we need to find the angle that B makes with respect to A.

The magnitude of a cross product is equal to

 $|\vec{A} \times \vec{B}| = |A| |B| \sin \theta$

Substitute in the values for the magnitude of B and of the cross product

 $A^2 = A(2A)\sin\theta$

Solve for theta

 $1 = 2\sin\theta$ $\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^{\circ}$

Since θ is 30°, B must be parallel to the x-axis.

 $^{^\}dagger \mathrm{Problem}$ from Essential University Physics, Wolfson