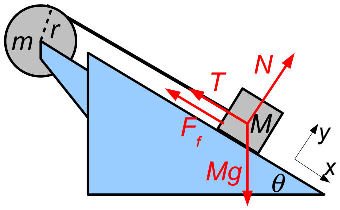
Chapter 10 Problem 57 †



Given
$\theta = 30°$
$M = 2.4\ kg$
$m = 0.85\ kg$
$r = 5.0\ cm = 0.050\ m$
$a = 1.6\ m/s^2$

Solution
Find the coefficient of friction between the ramp and the block.

Choose a coordinate system where the positive x-axis is parallel with the slope of the ramp and in the downhill direction. The y-axis is perpendicular to the ramp pointing upward. As the block slides down the ramp the force of friction and the tension in the string are exerting forces in the negative x direction. The weight of the block has a component in the positive x direction and in the negative y direction. The normal force exerted on the block by the ramp is in the positive y direction. Using Newton's 2nd law and resolving the forces on the sliding block in unit vector notation gives.

$$\Sigma \vec{F} = M\vec{a}$$

$$\vec{T} + \vec{F}_f + \vec{W} + \vec{N} = M\vec{a}$$

$$-T\hat{i} - F_f\hat{i} + Mg\sin\theta\hat{i} - Mg\cos\theta\hat{j} + N\hat{j} = Ma\hat{i}$$

Writing these as two scalar equations give

$$-T - F_f + Mg\sin\theta = Ma \tag{1}$$

$$-Mg\cos\theta + N = 0 \tag{2}$$

Since the string attaches the sliding block to the solid drum, the torque on the rotating drum is important. Assuming there is no friction opposing the rotation of the drum, the angular acceleration of the drum will be related to its moment of inertia and the torque applied to it. The moment of inertia for the drum would be equivalent to that of a solid disk. Therefore, the torque equation for the drum is

$$\tau = I\alpha$$

$$rT = \tfrac{1}{2}mr^2\alpha$$

Angular acceleration is related to linear acceleration by the equation $a = r\alpha$. Therefore, the torque equation becomes

$$rT = \tfrac{1}{2}mr^2(a/r)$$

†Problem from Essential University Physics, Wolfson

Dividing both sides by r and simplifying gives

$$T = \tfrac{1}{2}ma \tag{3}$$

Substitute equation (3) into equation (1) and solving for the force of friction gives

$$-(\tfrac{1}{2}ma) - F_f + Mg\sin\theta = Ma$$

$$-(\tfrac{1}{2}ma) + Mg\sin\theta - Ma = F_f$$

$$Mg\sin\theta - (\tfrac{1}{2}m + M)a = F_f$$

Substitute in the appropriate values gives a frictional force of

$$F_f = (2.4\ kg)(9.80\ m/s^2)\sin 30^\circ - \left(\tfrac{1}{2}(0.85\ kg) + (2.4\ kg)\right)(1.6\ m/s^2)$$

$$F_f = 11.76\ N - 4.52\ N = 7.24\ N$$

From equation (2) the normal force is

$$N = Mg\cos\theta = (2.4\ kg)(9.80\ m/s^2)\cos 30^\circ = 20.4\ N$$

The coefficient of friction between the block and the ramp is then

$$\mu = \frac{F_f}{N} = \frac{7.24\ N}{20.4\ N} = 0.35$$

The textbook says 0.36. The rounding is so close to breaking one way or the other, that my choice of $g = 9.80\ m/s^2$ resulted in it being rounded down.