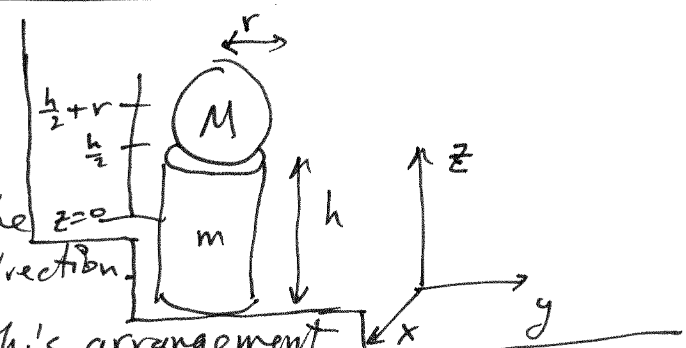
Chapter 9

Problem 76



a) Find the center of mass.

In this arrangement, the vertical is in the z-direction.

By the symmetry of this arrangement, the X_{cm} is lined up with the middle of the 2 objects. Likewise Y_{cm} is also at the middle of the objects.

However, in the z-direction we do not have ~~symmetry~~ symmetry.

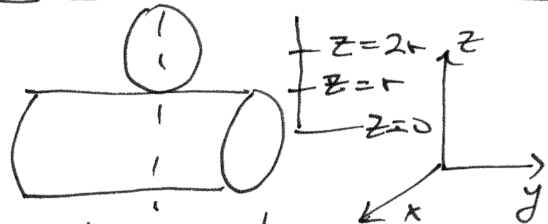
We are using the center of the cylinder as the origin. Then the mass of the cylinder is at $z_{cyl}=0$ and the mass of the sphere is at $z_{sph}=\frac{h}{2}+r$

Using the formula gives

$$Z_{cm}=\frac{\sum m_i z_i}{M}=\frac{m\cdot 0+M\left[\frac{h}{2}+r\right]}{m+M}$$

$$\boxed{Z_{cm}=\frac{M}{m+M}\left[\frac{h}{2}+r\right]} \qquad \boxed{\vec{r}_{cm}=0\hat{i}+0\hat{j}+\frac{M}{m+M}\left[\frac{h}{2}+r\right]\hat{k}}$$

b) Find the center of mass.



As explained above, the X_{cm} + Y_{cm} are lined up with the middle of the cylinder by symmetry.

(To explain further, if you put a mirror plane ~~at the~~ through the origin and aligned the plane with the x+z axis, you would find that what ~~is~~ exists from $0<y<\infty$ is a mirror image of what exists from $-\infty<y<0$.

This is also true by setting up a y-z mirror plane through the origin.

~~For~~ It is impossible to set up an x-y mirror plane for this object. Therefore, Z_{cm} is not at a symmetric middle.)

$$Z_{cm}=\frac{\sum m_i z_i}{M}=\frac{m\cdot 0+M\cdot 2r}{m+M}=\boxed{\frac{2Mr}{m+M}=Z_{cm}}$$