Chapter 7 Problem 106

Constant power is delivered to a car of mass m.

Show that if air resistance is ignored, the distance covered is $s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

If the engine produces constant power, then

$$P = \frac{dW}{dt} = \text{const} \rightarrow \underline{dW = P\,dt}$$

Now work is $W = \int F\,dx$ and $\underline{dW = F\,dx}$

Set these equations equal to each other

$$P\,dt = F\,dx \longrightarrow P = F\frac{dx}{dt} = F \cdot v$$

Since Power is a constant, then as velocity increases the force decreases. $P = F \cdot v$

Now $F = ma = m\frac{dv}{dt}$

and $P = m\frac{dv}{dt}v \longrightarrow P\,dt = m v\,dv$

Integrate both sides with $t_0 = 0$ + $v_0 = 0$ we have

$$\int_0^t P\,dt = \int_0^v m v\,dv \longrightarrow P \cdot t = \tfrac{1}{2} m v^2$$

Solving for v gives $v = \sqrt{\frac{2P \cdot t}{m}}$

But $v = \frac{ds}{dt} \longrightarrow ds = v\,dt = \sqrt{\frac{2P \cdot t}{m}}\,dt$

Integrate both sides with $s_0 = 0$ and $t_0 = 0$

$$\int_0^s ds = \int_0^t \sqrt{\frac{2Pt}{m}}\,dt = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2}\,dt = \sqrt{\frac{2P}{m}}\,\frac{t^{3/2}}{3/2} \Longrightarrow S = \sqrt{\frac{2P}{m\,\frac{3^2}{2^2}}} \cdot t^{3/2}$$

$$\boxed{S = \sqrt{\frac{8P}{[illegible]}}\, t^{3/2}}$$