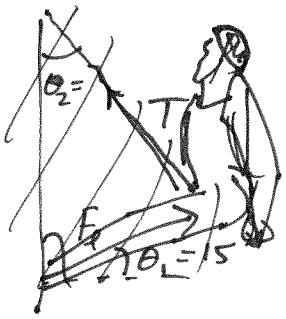
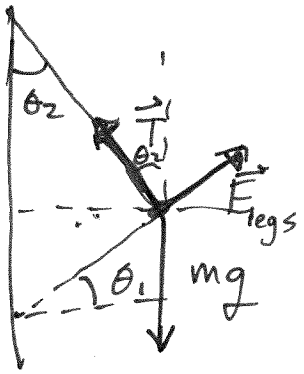
Chapter 6 Problem 61

$m = 52.0\ kg$ $\theta_1 = 15°$ $\theta_2 = 31°$



a) Find the tension in the rope and the force exerted on the vertical rock face.

Let x-axis be in the horizontal and y-axis in the vertical direction.



From Newton's 2nd Law

$$\Sigma \vec{F} = m\vec{a} = 0 \quad \text{(no motion of climber)}$$

$$\vec{T} + \vec{F}_{legs} + \vec{W} = 0$$

$$\vec{T} = -T\sin\theta_2\,\hat{i} + T\cos\theta_2\,\hat{j}$$

$$\vec{F}_{legs} = F_{legs}\cos\theta_1\,\hat{i} + F_{legs}\sin\theta_1\,\hat{j}$$

$$\vec{W} = -mg\,\hat{j}$$

$$-T\sin\theta_2\,\hat{i} + T\cos\theta_2\,\hat{j} + F_{legs}\cos\theta_1\,\hat{i} + F_{legs}\sin\theta_1\,\hat{j} - mg\,\hat{j} = 0$$

x-dir:

$$-T\sin\theta_2 + F_{legs}\cos\theta_1 = 0 \quad (\#1)$$

y-dir:

$$T\cos\theta_2 + F_{legs}\sin\theta_1 - mg = 0 \quad (\#2)$$

Take (#1) and solve for F_{legs}

$$F_{legs}\cos\theta_1 = T\sin\theta_2$$

$$F_{legs} = T\frac{\sin\theta_2}{\cos\theta_1} \quad (\#3)$$

Substitute into (#2)

$$T\cos\theta_2 + \left[T\frac{\sin\theta_2}{\cos\theta_1}\right]\sin\theta_1 - mg = 0$$

$$T\left[\cos\theta_2 + \sin\theta_2\tan\theta_1\right] = mg$$

$$T = \frac{mg}{\cos\theta_2 + \sin\theta_2\tan\theta_1} = \frac{(52.0\,kg)(9.80\,m/s^2)}{\cos(31°) + \sin(31°)\tan(15°)}$$

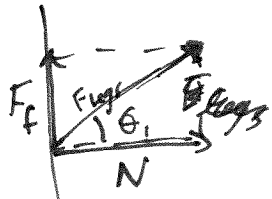
$$T = \frac{509.6\,N}{0.995} = \boxed{512\,N}$$

substitute this value into (#3)

$$F_{legs} = T\frac{\sin\theta_2}{\cos\theta_1} = (512N)\frac{\sin(31^\circ)}{\cos(15^\circ)} = \boxed{273N}$$

b) What is the minimum coefficient of friction between her shoes and the cliff.

If friction is holding her feet in place, then the vertical component of F_{legs} corresponds to the frictional force and the horizontal corresponds to the normal force.



From the diagram

$$N = F_{legs}\cdot\cos\theta_1$$
$$= (273N)\cos(15^\circ) = \underline{264N}$$

$$F_f = F_{legs}\cdot\sin\theta_1$$
$$= (273N)\sin(15^\circ) = 70.7N$$

Since $F_f = \mu N$ (maximum friction)

Then

$$\mu = \frac{F_f}{N} = \frac{70.7N}{264N} = \boxed{0.268}$$