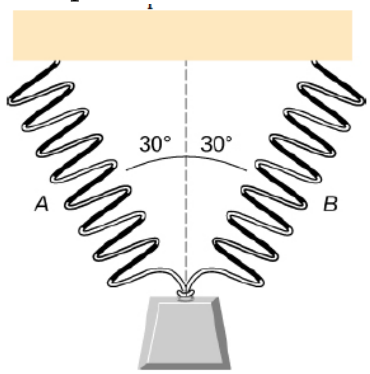
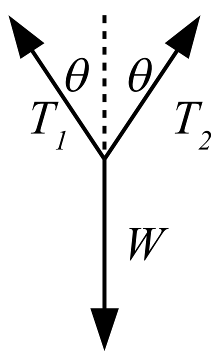
Chapter 5 Problem 87 †



Given
$k = 20\ N/m$
$W = 15.0\ N$

Solution
a) What is the tension in spring A?

The free-body diagram for this problem is



By Newton's 2nd law

$$\vec{F}_{net} = m\vec{a}$$

Substituting in the three forces acting on the system gives

$$\vec{T}_1 + \vec{T}_2 + \vec{W} = m\vec{a} = 0$$

Resolve each of the forces into horizontal (x-direction) and vertical (y-direction) coordinates. Since the tension is the same in each spring, $T_1 = T_2 = T$.

$$\vec{T}_1 = -T\sin(30°)\hat{\imath} + T\cos(30°)\hat{\jmath}$$

$$\vec{T}_2 = T\sin(30°)\hat{\imath} + T\cos(30°)\hat{\jmath}$$

†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

$$\vec{W} = -W\hat{\jmath}$$

Inserting these values into Newtwon's 2nd Law gives

$$-T\sin(30°)\hat{\imath} + T\cos(30°)\hat{\jmath} + T\sin(30°)\hat{\imath} + T\cos(30°)\hat{\jmath} - W\hat{\jmath} = 0$$

The x-component of this equation is

$$-T\sin(30°) + T\sin(30°) = 0$$

The y-component of this equation is

$$T\cos(30°) + T\cos(30°) - W = 0$$

The first equation is not useful; however, the second equation can be used to solve for the tension in the springs.

$$2T\cos(30°) = W$$

$$T = \frac{W}{2\cos(30°)}$$

Substitute in the given values

$$T = \frac{15.0\ N}{2\cos(30°)}$$

$$T = 8.66\ N$$

b) What is the stretch of each spring?

Hooke's Law states

$$F = -k\Delta x$$

The tension of the spring is in the upward direction. Therefore, the stretch of the spring is in the negative direction. Dropping the negative sign, the magnitude of the stretch is

$$\Delta x = \frac{F}{k}$$

$$\Delta x = \frac{8.66\ N}{20\ N/m} = 0.433\ m$$