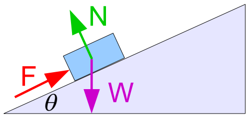
Chapter 5 Problem 67 [†]



Given
$m = 2.0\ kg$
$\theta = 30°$

Solution
a) Find the acceleration of the block and the force of the ramp on the block.

Using Newton's 2nd Law and substituting in each of the forces from the diagram gives

$$\Sigma \vec{F}_i = m\vec{a}$$

$$\vec{F} + \vec{N} + \vec{W} = m\vec{a} \qquad Eq(1)$$

We now need to resolve the vectors into a coordinate system. I will chose a system where the positive x-axis is pointing up the slope and the positive y-axis is pointing perpendicular upward from the ramp. In this coordinate system the acceleration can only be in the x-direction. Therefore,

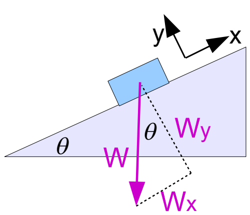
$$\vec{a} = a\hat{i}$$

The Force pushing on the block along the ramp is

$$\vec{F} = F\hat{i}$$

The normal force is in the positive y-direction.

$$\vec{N} = N\hat{j}$$



The weight is a combination of forces in the negative x-direction and the negative y-direction. By using geometry, you should be able to prove that the two angles labeled θ are the same angle. In the dotted triangle, notice that the x-component of the weight is opposite of the angle θ. Therefore, the sine function should be used. For the same reason, cosine should be used to resolve the y-component. Therefore, the weight vector is

$$\vec{W} = -mg\sin\theta\hat{i} - mg\cos\theta\hat{j}$$

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

From equation (1), Newton's 2nd law for the block is then

$$F\hat{i} + N\hat{j} - mg\sin\theta\hat{i} - mg\cos\theta\hat{j} = ma\hat{i}$$

This vector equation contains two scalar equations. In the x-direction we have

$$F - mg\sin\theta = ma \qquad\qquad Eq(2)$$

and in the y-direction we have

$$N - mg\cos\theta = 0 \qquad\qquad Eq(3)$$

For the first part of this problem, the force applied upslope is equal to zero. We want to know the acceleration that is due only to gravity. From equation (2) we have

$$-mg\sin\theta = ma$$

$$a = \frac{-mg\sin\theta}{m} = -g\sin\theta$$
$$a = -(9.80\ m/s^2)\sin(30°) = -4.90\ m/s^2$$

The negative means that the block accelerates in the negative x-direction, which is downslope. From equation (3) we can find the normal force, which is the force the ramp exerts on the block.

$$N = mg\cos\theta$$

$$N = (2.0\ kg)(9.80\ m/s^2)\cos(30°)$$
$$N = 17.0\ N$$

b) What force applied upward along the ramp would allow the block to move with constant velocity?

If the block moves at constant velocity, then the acceleration is zero. Using equation (2) we can determine the magnitude of the force.

$$F - mg\sin\theta = 0$$

$$F = mg\sin\theta$$
$$F = (2.0\ kg)(9.80\ m/s^2)\sin(30°)$$
$$F = 9.80\ N$$