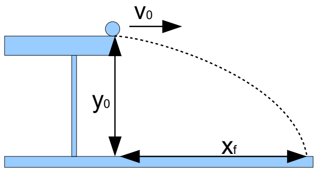
Chapter 4 Problem 34 [†]



Given
$y_0 = 1.0\ m$
$y_f = 0\ m$
$x_0 = 0\ m$
$x_f = 3.0\ m$
$g = 9.80\ m/s^2$
$\vec{a} = -g\hat{j} = -9.80\ \hat{j}\ m/s^2$

Solution
a) How long is the marble in the air?

Once the marble is in the air, the only force acting on it is gravity, which is in the negative $\hat{j}$ direction. The initial velocity in the y-direction is zero. Applying the 3rd kinematic equation in the y-direction we get

$$y_f = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$0 = y_0 + (0)t - \tfrac{1}{2}gt^2$$

Solving for time gives

$$\tfrac{1}{2}gt^2 = y_0$$

$$t = \sqrt{\frac{2y_0}{g}}$$

$$t = \sqrt{\frac{2(1.0\ m)}{9.80\ m/s^2}} = 0.452\ s$$

b) What is the speed of the marble when it leaves the table's edge?

Assuming there is no air drag slowing the marble down, the marble will move with constant speed in the x-direction. Therefore, acceleration in the x-direction is zero. Applying the third kinematic equation to the x-direction gives

$$x_f = x_0 + v_{x0}t - \tfrac{1}{2}a_x t^2$$

$$x_f = 0\ m + v_{x0}t - \tfrac{1}{2}(0)t^2$$

$$x_f = v_{x0}t$$

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Solving for the velocity in the x-direction gives

$$v_{x0} = \frac{x_f}{t} = \frac{3.0\ m}{0.452\ s} = 6.64\ m/s$$

Since there is no initial velocity in the y-direction, then initial velocity of the marble is

$$\vec{v}_0 = 6.6\ \hat{\imath}\ m/s$$

c) What is its speed when it hits the floor?

The speed in the y-direction can be found using the first kinematic equation.

$$v_{yf} = v_{y0} + a_y t = 0 - gt$$

$$v_{vf} = -(9.80\ m/s^2)(0.452\ s) = -4.43\ m/s$$

Using this result along with the velocity in the x-direction gives

$$\vec{v}_f = \{6.6\ \hat{\imath} - 4.4\ \hat{\jmath}\}\ m/s$$

The magnitude of this vector, which is speed, is

$$v_f = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.6\ m/s)^2 + (-4.4\ m/s)^2} = 7.9\ m/s$$