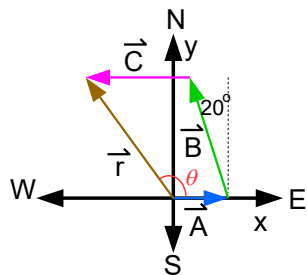


Chapter 4 Problem 21 †



Given

$$\vec{A} = 5.0 \hat{i} \text{ km}$$

$$\vec{B} = 10.0 \text{ km } \angle 20^\circ \text{ west of north}$$

$$\vec{C} = -8.0 \hat{i} \text{ km}$$

Solution

What is the final displacement of the cyclist?

First break vector B into scalar components using unit vectors. Since the angle is west of north, the opposite side of the angle corresponds to the negative x-axis. Therefore, the sine function is used to determine the x-component. The vector is pointing in the positive y-direction and this side corresponds to the adjacent side of the angle. Therefore, cosine is used for the y-component.

$$\vec{B} = \{-10.0 \sin(20^\circ) \hat{i} + 10.0 \cos(20^\circ) \hat{j}\} \text{ km}$$

$$\vec{B} = \{-3.42 \hat{i} + 9.40 \hat{j}\} \text{ km}$$

Adding the vectors together gives

$$\vec{r} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{r} = 5.0 \hat{i} \text{ km} + \{-3.42 \hat{i} + 9.40 \hat{j}\} \text{ km} - 8.0 \hat{i} \text{ km}$$

$$\vec{r} = \{-6.42 \hat{i} + 9.40 \hat{j}\} \text{ km}$$

The magnitude of this vector is

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(-6.42)^2 + (9.40)^2} = 11.4 \text{ km}$$

The angle of travel is

$$\theta = \tan^{-1} \left(\frac{r_y}{r_x} \right) = \tan^{-1} \left(\frac{9.40 \text{ km}}{-6.42 \text{ km}} \right) = -55.7^\circ$$

Since the x-component is negative and the y-component positive, the vector should be in the 2nd quadrant, not the 4th. Add 180° and the answer is

$$\theta = -55.7^\circ + 180^\circ = 124.3^\circ$$

If you want to express this angle with regard to the geography, then it is 56° north of west or 34° west of north.

†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)