Chapter 3 Problem 78 [†]

Given

$a(t) = pt^2 - qt^3$

Solution

a) What is the velocity as a function of time assuming initial velocity and initial position are both zero?

The definition of acceleration is

$$a = \frac{dv}{dt}$$

Separating the differentials gives

$$dv = adt$$

Substituting in the function for acceleration and setting up a definite integral on both sides gives

$$\int_0^v dv = \int_0^t a \cdot dt = \int_0^t (pt^2 - qt^3)dt$$

Completing the integration gives

$$v|_0^v = \left(\tfrac{1}{3}pt^3 - \tfrac{1}{4}qt^4\right|_0^t$$

$$v - 0 = \left(\tfrac{1}{3}pt^3 - \tfrac{1}{4}qt^4\right) - (0 - 0)$$

The velocity as a function of time is, therefore,

$$v(t) = \tfrac{1}{3}pt^3 - \tfrac{1}{4}qt^4$$

b) What is the position as a function of time assuming initial velocity and initial position are both zero?

The definition of velocity is

$$v = \frac{dx}{dt}$$

Separating the differentials gives

$$dx = vdt$$

Substituting in the function for velocity and setting up a definite integral on both sides gives

$$\int_0^x dx = \int_0^t v \cdot dt = \int_0^t \left(\tfrac{1}{3}pt^3 - \tfrac{1}{4}qt^4\right) dt$$

Completing the integration gives

$$x|_0^x = \left(\tfrac{1}{12}pt^4 - \tfrac{1}{20}qt^5\right|_0^t$$

$$x - 0 = \left(\tfrac{1}{12}pt^4 - \tfrac{1}{20}qt^5\right) - (0 - 0)$$

The position as a function of time is, therefore,

$$x(t) = \tfrac{1}{12}pt^4 - \tfrac{1}{20}qt^5$$

[†] Problem from University Physics by Ling, Sanny and Moebs (OpenStax)