Chapter 3 Problem 102 [†]

Given
$h = 300\ m$
$v_0 = 10.0\ m/s$
$a = -g = -9.80\ m/s^2$

Solution
a) Find the maximum height reached by the coin.

When the coin reaches max height, its velocity is zero. The time for this to happen can be calculated with the first kinematic equation.

$$v_f = v_0 + at$$

Solving for time gives

$$at = v_f - v_0$$

$$t = \frac{v_f - v_0}{a}$$

Assuming the upward direction is positive and downward is negative, then the acceleration due to gravity is in the negative direction. The initial velocity on the other hand is in the positive direction. Substituting in the appropriate values the time to max height is

$$t_1 = \frac{0\ m/s - 10.0\ m/s}{-9.80\ m/s^2} = 1.02\ s$$

Using the third kinematic equation, the maximum height is

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x = 300\ m + (10.0\ m/s)(1.02\ s) + \tfrac{1}{2}(-9.80\ m/s^2)(1.02\ s)^2$$

$$x = 300\ m + 10.2\ m - 5.1\ m = 305.1\ m$$

b) Find the coin's position after 4 seconds.

Since the time is known, use the third kinematic equation again.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x = 300\ m + (10.0\ m/s)(4.00\ s) + \tfrac{1}{2}(-9.80\ m/s^2)(4.00\ s)^2$$

$$x = 300\ m + 40.0m - 78.4\ m = 261.6\ m$$

c) Find the time to hit the ground.

This time can be calculated two ways. First, we could find the time for the coin to fall from the maximum height and add that time to the time found in part a. In this case the initial velocity is zero and the change in distance is the answer to part a. Start with the third kinematic equation.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

[†] Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Since $v_0 = 0$ and at ground level $x = 0$, we can solve for time.

$$0 = x_0 + 0 + \tfrac{1}{2}at^2$$

$$-x_0 = \tfrac{1}{2}at^2$$

$$\frac{-2x_0}{a} = t^2$$

$$t = \sqrt{\frac{-2x_0}{a}}$$

Substituting in the maximum height of the coin gives a time of

$$t_2 = \sqrt{\frac{-2(305.1\ m)}{-9.80\ m/s^2}} = 7.89\ s$$

Add to this the time it took to get to maximum height and you have the time for the coin to reach the ground.

$$t_{total} = t_1 + t_2 = 1.02\ s + 7.89\ s = 8.91\ s$$

Total time is 8.91 s.

A second way of solving this problem is to use the third kinematic equation and use the initial values given in the problem. In this case $x_0 = 300\ m$, $v_0 = 10.0\ m/s$ and at ground level $x = 0\ m$.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$0 = 300\ m + (10.0\ m/s)t + \tfrac{1}{2}(-9.80\ m/s^2)t^2$$

Dropping the units for clarity, we have an equation that can be solved with the quadratic equation.

$$0 = 300 + 10.0t - 4.90t^2$$

Remember that a quadratic equation of the form $Ax^2 + Bx + C = 0$ has solutions given by the equation

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Our equation uses time, t instead of x. Comparing the two equations, $A = -4.90$, $B = 10.0$ and $C = 300$. Our solution is then

$$t = \frac{-10.0 \pm \sqrt{(10.0)^2 - 4(-4.90)(300)}}{2(-4.90)}$$

$$t = \frac{10.0 \mp \sqrt{100 + 5880}}{9.80}$$

$$t = \frac{10.0 \mp 77.3}{9.80}$$

The two solutions are

$$t = -6.87\ s, \qquad 8.91\ s$$

The solutions to the quadratic equation are just looking for times when the parabolic shape intersects the ground. The negative solution corresponds to a projectile fired from the ground in an upward direction. 6.87 s after it is fired, it is at the height of the balloon and rising with a velocity of 10.0 m/s. Running this scenario backwards from the balloon gives a negative time to get to the surface.

The second solution is the time for the projectile to return to the surface from the location of the balloon. This second solution is the answer to our problem. Notice this second method gives the same answer as the first method.