Chapter 2 Problem 66[†]

Given $\vec{D} = \{2.0\hat{i} - 4.0\hat{j} + \hat{k}\} N$ $\vec{G} = \{3.0\hat{i} + 4.0\hat{j} + 10.0\hat{k}\} N$

Solution

Show that the two force vectors are orthogonal.

If something is orthogonal, it means their directions are 90° away from each other. Now we know that the polar form of the dot product is

 $\vec{D} \cdot \vec{G} = \|\vec{D}\| \|\vec{G}\| \cos \theta$

In Cartesian form using unit vectors, you get

$$\vec{D} \cdot \vec{G} = D_x G_x + D_y G_y + D_z G_z$$

Setting these equal to each other, we get

$$\|\vec{D}\|\|\vec{G}\|\cos\theta = D_xG_x + D_yG_y + D_zG_z$$

If the vectors are orthogonal, then $\theta = 90^{\circ}$ and $\cos \theta = \cos 90^{\circ} = 0$. Therefore, the right side of the equation must also equal zero. Performing the dot product in cartesian coordinates gives

 $D_xG_x + D_yG_y + D_zG_z = (2.0)(3.0) + (-4.0)(4.0) + (1.0)(10.0) = 6 - 16 + 10 = 0$

Since the dot product equals zero, the two vectors are orthogonal.

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)