Chapter 1 Problem 86 [†]

Given
$length = 3.955 \pm 0.005\ m$
$width = 3.050 \pm 0.005\ m$

Solution
Find the area and uncertainty of the area for the room.

Area is length times width.

$$A = l \cdot w = (3.955\ m)(3.050\ m) = 12.06275\ m^2$$

We should expect the answer to be good to four sig.figs. That would give us

$$A = 12.06\ m^2$$

For a rough estimate of error, we could assume the last digit is off by 1, thus giving $\pm 0.01\ m^2$. However, a more accurate estimate of error is to add the percentage of error for the length and width.

$$length\ percent\ error = \frac{\Delta\ l}{l} \times 100\% = \frac{.005\ m}{3.955\ m} \times 100\% = 0.126\ \%$$

$$width\ percent\ error = \frac{\Delta\ w}{w} \times 100\% = \frac{.005\ m}{3.050\ m} \times 100\% = 0.164\ \%$$

Therefore,

$$area\ percent\ error = 0.126\% + 0.164\ \% = 0.290\ \%$$

The uncertainty in the area is then

$$\Delta\ A = A \left(\frac{area\ percent\ error}{100\ \%}\right)$$

$$\Delta\ A = (12.06275\ m^2) \left(\frac{0.29\ \%}{100\ \%}\right) = 0.035\ m^2$$

The area of the room is

$$A = 12.06 \pm 0.04\ m^2.$$

Notice the answer really is good to four significant digits, but the uncertainty is a bit larger than our rough estimate.

Also notice, that the measurement error is 1/2 cm. If the room were 1/2 cm larger in both length and width, the new area would be 12.0978 m^2. This value is 0.03505 larger than our calculated value, which is right in line with our calculated uncertainty. You could do the same calculation assuming the length and width are 1/2 cm smaller than the given value. You will get a difference of comparable value.

[†] Problem from University Physics by Ling, Sanny and Moebs (OpenStax)