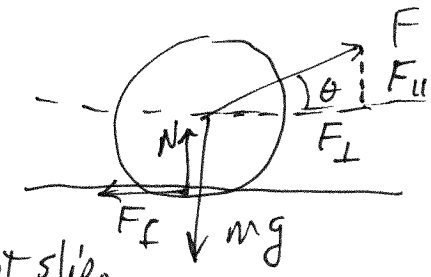
$\theta = 37°$

$\mu_s = 0.40 \quad \mu_k = 0.30$




What is the maximum value of F where the wheel does not slip.

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By Newton's 2nd Law

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{F}_f + \vec{N} + \vec{W} + \vec{F} = m\vec{a}$$

$$-F_f \hat{i} + N\hat{j} - mg\hat{j} + F\cos\theta \hat{i} + F\sin\theta \hat{j} = ma\hat{i}$$

Break into 2 equations

x-dir $-F_f + F\cos\theta = ma$ (#1)

y-dir $N - mg + F\sin\theta = 0$ (#2)

also $F_f = \mu N$ (#3) (where $\mu$ is the static coefficient of friction. The wheel is rolling rather than sliding over the surface.)

We have 3 equations + 4 unknowns

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Using the torque equation $\Sigma \tau = I\alpha$

$\tau_N = 0$ because it acts at the pivot ~~point~~ point (contact between road + wheel)

$\tau_{F_g} = 0$ " " " " " "

$\tau_W = 0$ because ~~the~~ The angle between $\vec{r}$ + $\vec{F}$ is 180°

only torque is due to F. $F_\perp$ to $\vec{r}$

~~$\tau = RF\sin$~~

The perpendicular portion of $\vec{F}$ to $\vec{r}$ is $F\cos\theta$

$\therefore \quad \tau = R F_\perp = R F\cos\theta$

$\tau = I\alpha \longrightarrow I\alpha = RF\cos\theta$ (#4)

The moment of inertia of the wheel about the pivot point is

$$I = I_{cm} + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

($I_{cm}$: solid cylinder; $h$: radius of wheel)

Substitute this into #4 + convert using $\alpha = \frac{a}{R}$

$$I\alpha = RF\cos\theta$$

$$\frac{3}{2}MR^2\left(\frac{a}{R}\right) = RF\cos\theta \rightarrow \frac{3}{2}MRa = RF\cos\theta$$

$$a = \frac{2}{3}\frac{F}{M}\cos\theta$$

Substitute this into #1

$$-F_f + F\cos\theta = M\left[\frac{2}{3}\frac{F}{M}\cos\theta\right]$$

combine #2 + #3

$$N = mg - F\sin\theta \rightarrow F_f = \mu[mg - F\sin\theta]$$

Then

$$-\mu[mg - F\sin\theta] + F\cos\theta = \frac{2}{3}F\cos\theta$$

$$-\mu mg + \mu F\sin\theta + F\cos\theta = \frac{2}{3}F\cos\theta$$

Rearrange terms

$$\mu F\sin\theta + F\cos\theta - \frac{2}{3}F\cos\theta = \mu mg$$

$$F\left[\mu\sin\theta + \cos\theta - \frac{2}{3}\cos\theta\right] = \mu mg$$

$$F\left[\mu\sin\theta + \frac{1}{3}\cos\theta\right] = \mu Mg$$

$$F = \frac{\mu Mg}{\mu\sin\theta + \frac{1}{3}\cos\theta} = \frac{(0.40)Mg}{0.40\sin(37^\circ) + \frac{1}{3}\cos(37^\circ)}$$

$$\boxed{F = 0.79Mg}$$