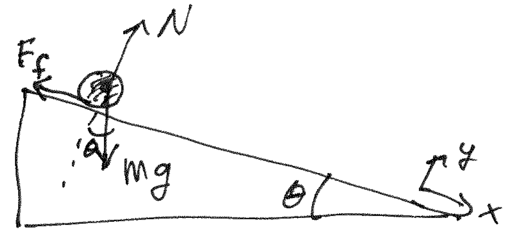
Marble rolls down an incline from rest.



$v_0 = 0\,m/s \qquad \theta = 30°$

a) What is its acceleration?

Moment of inertia for a solid sphere is

$$I_{cm} = \frac{2}{5} M R^2$$ (Figure 10.20 - Textbook p. 497)

From the free-body diagram there are 3 forces.

from Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_f + \vec{N} + \vec{W} = m\vec{a}$$

$$-F_f\hat{i} + N\hat{j} + mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j} = ma\,\hat{i}$$

Break into x & y directions gives

x) $-F_f + mg\sin\theta = ma$ #1

y) $N - mg\cos\theta = 0$ #2

This gives us 2 equations with 3 unknowns.

Before we invoked our definition of friction

$$F_f \leq \mu N$$

However, in this case we only use enough friction to keep the marble from slipping on the ramp.

Therefore, this inequality is not useful

Our 3rd equation comes from the rotational equivalent to Newton's 2nd Law.

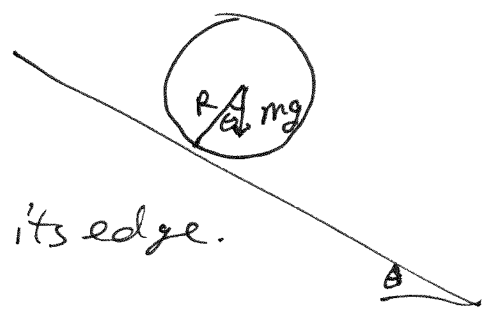
$$\sum \tau_i = I\alpha \longrightarrow \tau_{FR} + \tau_N + \tau_W = I\alpha$$

Remember that $\tau = rF\sin\theta$

If we treat the point where the marble makes contact with the ~~ramp~~ ramp as the pivot point, then

- Friction acts at $r=0$ $\therefore \tau_{FR} = 0$
- Normal force acts at $r=0$ $\therefore \tau_N = 0$
- Weight acts at $r=R$ with an angle θ between $\vec{r}$ + $\vec{F}$

$\therefore \tau_W = Rmg\sin\theta$



Since $\tau = I\alpha$, then

$$Rmg\sin\theta = I\alpha$$

I is now that of a sphere about its edge.

Use the parallel-axis theorem

$$I = I_{cm} + mh^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

Also $\alpha = \frac{a}{r} \longrightarrow Rmg\sin\theta = \frac{7}{5}MR^2\frac{a}{R}$ ✱

So $\boxed{a = \frac{5g\sin\theta}{7}}$ ✱ Notice: we have 1 equation with 1 unknown

Then $a = \frac{5}{7}(9.8\,m/s^2)\sin 30^\circ = \boxed{3.5\,m/s^2}$

b) How far does it go in 3.0 s

Using the 3rd kinematic equation

$$X = X_0 + V_0t + \tfrac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(3.5\,\frac{m}{s^2}\right)(3.0s)^2$$

$$X = 15.75\ m$$

$$\boxed{15.8\,m}$$