Chapter 9 Problem 36 †

Given

Three equal masses

$\vec{r}_1 = \{(3t^2 + 5)\hat{\imath}\}$

$\vec{r}_2 = \{(7t + 2)\hat{\imath} + 2\hat{\jmath}\}$

$\vec{r}_3 = \{(3t)\hat{\imath} + (2t + 6)\hat{\jmath}\}$

Solution

a) Find the position of the center of mass.

Assume that each of the masses have a value of 1 kg. Then the total mass is

$$M = 3\ kg$$

The position of the center of mass is then

$$\vec{R} = \frac{\Sigma m_i \vec{r}_i}{M} = \frac{m \Sigma \vec{r}_i}{M}$$

$$\vec{R} = \frac{(1\ kg)\left(\{(3t^2 + 5)\hat{\imath}\} + \{(7t + 2)\hat{\imath} + 2\hat{\jmath}\} + \{(3t)\hat{\imath} + (2t + 6)\hat{\jmath}\}\right)}{3\ kg}$$

$$\vec{R} = \left\{(t^2 + \tfrac{10}{3}t + \tfrac{7}{3})\hat{\imath} + (\tfrac{2}{3}t + \tfrac{8}{3})\hat{\jmath}\right\}$$

b) Find the velocity of the center of mass.

From the position of the center of mass, take the first derivative wrt. time and get the velocity.

$$\vec{V} = \frac{d\vec{R}}{dt} = \frac{d\left\{(t^2 + \tfrac{10}{3}t + \tfrac{7}{3})\hat{\imath} + (\tfrac{2}{3}t + \tfrac{8}{3})\hat{\jmath}\right\}}{dt}$$

$$\vec{V} = \left\{(2t + \tfrac{10}{3})\hat{\imath} + (\tfrac{2}{3})\hat{\jmath}\right\}$$

c) Find the acceleration of the center of mass.

From the velocity of the center of mass, take the first derivative wrt. time and get the acceleration.

$$\vec{A} = \frac{d\vec{V}}{dt} = \frac{d\left\{(2t + \tfrac{10}{3})\hat{\imath} + (\tfrac{2}{3})\hat{\jmath}\right\}}{dt}$$

$$\vec{A} = 2\hat{\imath}$$

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†Problem from Essential University Physics, Wolfson