Chapter 7 Problem 52 [†]

Given
$U = ax^2 - bx + c$
$a = 5.20\ N/m$
$b = 3.12\ N$
$c = 0.468\ J$

Solution
a) Find the equilibrium point of the spring.

The equilibrium point will be where there is no force exerted by the spring. The force of the spring in the x direction is

$$F_x = -\frac{dU}{dx} = -\frac{d(ax^2 - bx + c)}{dx} = -2ax + b \tag{1}$$

Set the F_x equal to zero and solve for x.

$$0 = -2ax + b$$

$$x = \frac{b}{2a}$$

Substitute in the appropriate values for a and b gives

$$a = \frac{(3.12\ N)}{2(5.20\ N/m)} = 0.30\ m = 30\ cm$$

b) Find the spring constant.

From Hooke's law the force of the spring is

$$F = -k\Delta x = -k(x - x_0)$$

Getting equation (1) into this form gives

$$F_x = -2ax + b = -2a\left(x - \frac{b}{2a}\right)$$

Therefore, $k = 2a$. Substituting in the appropriate value gives

$$k = 2(5.20\ N/m) = 10.4\ N/m$$

[†]Problem from Essential University Physics, Wolfson