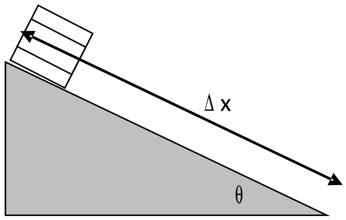
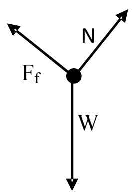
**Chapter 5 Problem 55** †




**Given**
$\Delta x = 5.4\ m$
$\theta = 30°$
$t = 3.3\ s$
$v_0 = 0\ m/s$

**Solution**
Free-body diagram of the crate.




Find the maximum coefficient of friction that allows the crate to reach the bottom of the ramp in at least 3.3 $s$.

The slope of the ramp and the coefficient of friction are constant; therefore, the acceleration of the crate down the ramp will also be constant.
Applying the kinematic equations to the problem, find the acceleration of the crate.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Using the initial velocity of 0 $m/s$ and an initial position of 0 $m$, the acceleration is

$$x = \tfrac{1}{2}at^2$$

$$a = \frac{2x}{t^2}$$

$$a = \frac{2(5.4\ m)}{(3.3\ s)^2} = 0.99\ m/s^2$$

From the free-body diagram given above and using Newton's $2^{nd}$ law gives

$$\Sigma\vec{F} = m\vec{a}$$

$$\vec{F}_f + \vec{W} + \vec{N} = m\vec{a}$$

Choose the coordinate system so that the x axis is down the ramp. The acceleration will be in the x direction. The weight of the crate in this coordinate system is

$$\vec{W} = mg\sin\theta\hat{i} - mg\cos\theta\hat{j}$$

†Problem from Essential University Physics, Wolfson

Writing out Newton's $2^{nd}$ law in unit vector notation gives

$$-\mu_k N\hat{i} + mg\sin\theta\hat{i} - mg\cos\theta\hat{j} + N\hat{j} = ma\hat{i}$$

The x-component of this equation is

$$-\mu_k N + mg\sin\theta = ma \tag{1}$$

and the y-component of this equation is

$$-mg\cos\theta + N = 0 \tag{2}$$

From equation (2) we see that the normal force is $mg\cos\theta$. Substitute this into equation(1) and solve for the acceleration.

$$-\mu_k mg\cos\theta + mg\sin\theta = ma$$

$$\mu_k = \frac{mg\sin\theta - ma}{mg\cos\theta}$$

$$\mu_k = \tan\theta - \frac{a}{g\cos\theta}$$

$$\mu_k = \tan(30^\circ) - \frac{0.99\ m/s^2}{(9.8\ m/s^2)\cos 30^\circ} = 0.46$$

Therefore, the coefficient of friction must be less than or equal to 0.46.