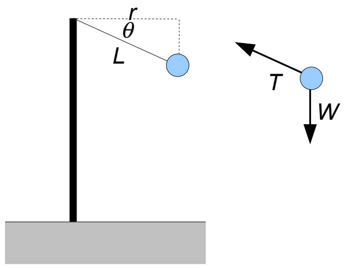
Chapter 5 Problem 26 †



Given
$L = 1.7\ m$
$\theta = 15°$

Solution
Find the speed of the tetherball.

Using the free-body diagram given above and Newton's 2^{nd} law gives

$$\Sigma\vec{F} = m\vec{a}$$

$$\vec{T} + \vec{W} = m\vec{a}$$

Making the x direction horizontal and resolving into unit vector notation gives

$$-T\cos\theta\hat{i} + T\sin\theta\hat{j} - mg\hat{j} = m\vec{a}$$

Now the ball is experiencing centripetal acceleration because it is moving at constant speed at a radius of r. This acceleration is in the $-x$ direction. The equation becomes

$$-T\cos\theta\hat{i} + T\sin\theta\hat{j} - mg\hat{j} = -ma\hat{i}$$

The x-component equation is

$$-T\cos\theta = -ma \tag{1}$$

and the y-component equation is

$$T\sin\theta - mg = 0 \tag{2}$$

Use equation (2) and solve for T.

$$T = \frac{mg}{\sin\theta}$$

Substitute this into equation (1) and solve for a.

$$-\left(\frac{mg}{\sin\theta}\right)\cos\theta = -ma$$

$$\frac{mg}{\tan\theta} = ma$$

†Problem from Essential University Physics, Wolfson

$$a = \frac{g}{\tan\theta} = \frac{(9.8\ m/s^2)}{\tan(15^\circ)} = 36.6\ m/s^2$$

Since the acceleration is centripetal, then

$$a = \frac{v^2}{r}$$

Solving for speed gives

$$v^2 = ar$$

$$v = \sqrt{ar} \tag{3}$$

From the diagram notice that r is the adjacent side of the given angle; therefore,

$$r = L\cos\theta$$

and equation (3) becomes

$$v = \sqrt{aL\cos\theta} = \sqrt{(36.6\ m/s^2)(1.7\ m)\cos(15^\circ)} = 7.75\ m/s$$