

Chapter 3 Problem 60 †

Given

$$\vec{v}_0 = \{11\hat{i} + 14\hat{j}\} \text{ m/s}$$

$$\vec{a} = \{-1.2\hat{i} + 0.26\hat{j}\} \text{ m/s}^2$$

Particle begins at the origin.

Solution

a) When does the particle cross the y axis?

The position vector of the particle is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{r} = 0 + \{11\hat{i} + 14\hat{j}\} t + \frac{1}{2} \{-1.2\hat{i} + 0.26\hat{j}\} t^2$$

Regrouping gives

$$\vec{r} = \{(11t - 0.60t^2)\hat{i} + (14t + 0.13t^2)\hat{j}\} \text{ m}$$

The particle crosses the y axis when the x component equals zero.

$$11t - 0.60t^2 = 0$$

$$(11 - 0.60t)t = 0$$

The solutions to this equation are $t = 0 \text{ s}$ and $t = 18.3 \text{ s}$. The first solution is the initial condition of the problem. The second solution is when it crosses back again. Therefore,

$$t = 18.3 \text{ s}$$

b) What is the y coordinate at this time?

Take the y component of the position vector and substitute in $t = 18.3 \text{ s}$.

$$y = 14(18.3) + 0.13(18.3)^2 = 300 \text{ m}$$

c) Find the speed and direction of the particle at this time.

The velocity at $t = 18.3 \text{ s}$ is

$$\vec{v} = \vec{v}_0 + \vec{a}t = \{11\hat{i} + 14\hat{j}\} \text{ m/s} + \{-1.2\hat{i} + 0.26\hat{j}\} \text{ m/s}^2(18.3 \text{ s})$$

$$\vec{v} = \{-11.0\hat{i} + 18.8\hat{j}\} \text{ m/s}$$

The magnitude of this vector is

$$v = \sqrt{(-11.0)^2 + (18.8)^2} \text{ m/s}$$

$$v = 21.8 \text{ m/s}$$

The direction of this vector is in the second quadrant.

$$\theta = \tan^{-1} \left(\frac{18.8}{-11.0} \right) = 120^\circ.$$

†Problem from Essential University Physics, Wolfson