Chapter 13 Problem 34 [†]

Given

$$m = 1400 \ kg$$

 $v = 20 \ m/s$
 $f = 0.67 \ Hz$
 $A = 18 \ cm = 0.18 \ m$

Solution

a) Find the total energy in the oscillations.

For a harmonic oscillator, the velocity is the first derivative of the position function. This results in a maximum velocity of $A\omega$. The kinetic energy at this maximum velocity is

$$U_k = \frac{1}{2}mv^2 = \frac{1}{2}m(A\omega)^2$$

Since energy in an oscillation goes between potential and kinetic energy, this kinetic energy at maximum velocity is also the energy present in the oscillation. Remember that at maximum velocity the potential energy of the oscillation is zero. Therefore, the energy in the oscillation is

$$U_k = \frac{1}{2}(1400 \ kg)((0.18 \ m)2\pi(0.67 \ Hz))^2$$

$$U_k = 402 \ J$$

(Note: We need angular frequency for this formula. In this problem frequency was given. Therefore, when solving this problem, we replace ω with $2\pi f$.)

b) Find the fraction of the car's kinetic energy in the oscillations.

$$E_{total} = E_{trans} + U_k = \frac{1}{2}mv^2 + U_k$$
$$E_{total} = \frac{1}{2}(1400 \ kg)(20 \ m/s)^2 + 402 \ J = 280400 \ J$$

% energy is then

%energy =
$$\frac{U_k}{E_{total}} \times 100 \% = \frac{(402 \ J)}{(280400 \ J)} \times 100 \%$$

% energy = 0.143 %

[†]Problem from Essential University Physics, Wolfson