

Chapter 13 Problem 24 †

Given

$$f = 32,768 \text{ Hz}$$

$$A = 100 \text{ nm} = 1.0 \times 10^{-7} \text{ m}$$

Solution

Find the maximum velocity and acceleration of the face of the quartz crystal.

The position of the quartz crystal face is given by

$$x = A \cos(\omega t + \phi)$$

The maximum displacement is the amplitude, A . The angular frequency, ω , is calculated from the frequency using the relationship $\omega_0 = 2\pi f$.

Taking the first derivative of the position function with respect to time gives the velocity,

$$v = -A\omega \sin(\omega t + \phi)$$

The maximum velocity is the amplitude times the angular frequency. Substituting in the appropriate values give a maximum velocity of

$$v_{\max} = A\omega = A2\pi f = (1.0 \times 10^{-7} \text{ m})2\pi(32,768 \text{ Hz})$$

$$v_{\max} = 2.06 \times 10^{-2} \text{ m/s} = 2.06 \text{ cm/s}$$

Taking the derivative of the velocity function gives the acceleration,

$$a = -A\omega^2 \cos(\omega t + \phi)$$

The maximum velocity is the amplitude times the angular frequency squared. Substituting in the appropriate values give a maximum acceleration of

$$a_{\max} = A\omega^2 = A(2\pi f)^2 = (1.0 \times 10^{-7} \text{ m})(2\pi(32,768 \text{ Hz}))^2$$

$$a_{\max} = 4.24 \times 10^3 \text{ m/s}^2$$

†Problem from Essential University Physics, Wolfson