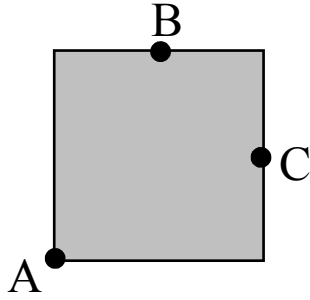


Chapter 12 Problem 18 †



Solution

Find the gravitational torque about each of the indicated points on a square plate of uniform thickness.

The gravitational force is always downward from the center of the plate with a value of

$$\vec{F}_g = -mg\hat{j}$$

The position vector of the center of gravity with respect to point A is

$$\vec{r} = \left\{ \frac{1}{2}l\hat{i} + \frac{1}{2}l\hat{j} \right\}$$

The gravitational torque about point A is

$$\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{\tau} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2}l & \frac{1}{2}l & 0 \\ 0 & -mg & 0 \end{vmatrix} \\ \vec{\tau} &= -\frac{1}{2}mgl\hat{k} \end{aligned}$$

The position vector of the center of gravity with respect to point B is

$$\vec{r} = \left\{ -\frac{1}{2}l\hat{j} \right\}$$

The gravitational torque about point B is

$$\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{\tau} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -\frac{1}{2}l & 0 \\ 0 & -mg & 0 \end{vmatrix} \\ \vec{\tau} &= 0 \end{aligned}$$

The position vector of the center of gravity with respect to point C is

$$\vec{r} = \left\{ -\frac{1}{2}l\hat{i} \right\}$$

The gravitational torque about point C is

$$\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{\tau} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{2}l & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix} \\ \vec{\tau} &= \frac{1}{2}mgl\hat{k} \end{aligned}$$

†Problem from Essential University Physics, Wolfson