

Chapter 11 Problem 14 †

Given

$$\begin{aligned}\omega_0 &= 45 \hat{j} \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 4.71 \hat{j} \text{ rad/s} \\ \omega_f &= 60 \hat{j} \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 6.28 \hat{j} \text{ rad/s} \\ t &= 15 \text{ s}\end{aligned}$$

Solution

a) Find the magnitude of the average angular acceleration.

The average angular acceleration is

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t} = \frac{\vec{\omega}_f - \vec{\omega}_0}{\Delta t} = \frac{6.28 \hat{i} - 4.71 \hat{j}}{15} \text{ rad/s}^2$$

$$\vec{\alpha} = \{0.419 \hat{i} - 0.314 \hat{j}\} \text{ rad/s}^2$$

The magnitude of this angular acceleration is

$$\alpha = \sqrt{\alpha_x^2 + \alpha_y^2} = \sqrt{(0.419 \text{ rad/s}^2)^2 + (-0.314 \text{ rad/s}^2)^2}$$

$$\alpha = 0.524 \text{ rad/s}^2$$

b) Find the angle of this angular acceleration with respect to horizontal.

The two components of the angular acceleration give an angle of

$$\theta = \tan^{-1} \left(\frac{\alpha_y}{\alpha_x} \right) = \tan^{-1} \left(\frac{-0.314}{0.419} \right) = -36.8^\circ$$

†Problem from Essential University Physics, Wolfson