Chapter 10 Problem 55 †

Given
$D = 22\ m$
$m = 5.0 \times 10^5\ kg$
$F = 100\ N$

Solution
a) To simulate one gravity how long will it take the rockets to get the desired spin rate.

In order to simulate one gravity, the centripetal acceleration must be

$$g = a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

Solving for the angular velocity gives

$$\omega = \sqrt{\frac{g}{r}}$$

Since there are two rockets providing a force at a radius of 11 m, the torque is

$$\tau = 2rF$$

This torque results in an angular acceleration.

$$\tau = I\alpha$$

Solving for angular acceleration and substituting in the relationship between torque and force gives

$$\alpha = \frac{\tau}{I} = \frac{2rF}{I} = \frac{2rF}{mr^2} = \frac{2F}{mr}$$

The moment of inertia for the space station is mr^2 since the mass is concentrated at the rim. We now have an initial velocity of zero, a final velocity, and an acceleration for rotational motion. These quantities are related by

$$\omega_f = \omega_0 + \alpha t$$

Solving for time gives

$$t = \frac{\omega_f - \omega_0}{\alpha} = \frac{\sqrt{g/r} - 0}{\left(\frac{2F}{mr}\right)} = \frac{mr}{2F}\sqrt{\frac{g}{r}}$$

Substituting in the given values from the problem gives a times of

$$t = \frac{(5.0 \times 10^5\ kg)(11\ m)}{2(100\ N)}\sqrt{\frac{(9.80\ m/s^2)}{(11\ m)}} = 2.60 \times 10^4\ s$$

$$t = 7.21\ h$$

b) Find the number of revolutions made by the station during this time.

†Problem from Essential University Physics, Wolfson

The angle rotated through is given by the formula

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

The initial angular velocity is zero so the angle becomes

$$\Delta\theta = \tfrac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2F}{mr}\right)t^2$$

$$\Delta\theta = \frac{1}{2}\left(\frac{2(100\ N)}{(5.0 \times 10^5\ kg)(11\ m)}\right)(2.60 \times 10^4\ s)^2$$

$$\Delta\theta = 1.23 \times 10^4\ rad$$

Convert this into revolutions by dividing by 2π gives

$$\Delta\theta = 1960\ rev$$