Contact Ratio

\[ m_p = \frac{Z}{p_b} \text{ (length of action) \over p_b} \text{ (base pitch) \over \pi \cos \phi} = \frac{Pd}{\pi \cos \phi} \]

To find the length of action, consider the distance along the line of contact from the pitch point to the addendum circle.

\[ b = r_0 \cos \phi \]
\[ g = r_0 \sin \phi \]
\[ f = \sqrt{(r_0+a)^2 - b^2} \]
\[ f = -\sqrt{(r_0+a)^2 - r_0^2 \cos^2 \phi} \]
\[ z_G = f - g \]
\[ z_G = -\sqrt{(r_0+a)^2 - r_0^2 \cos^2 \phi} - r_0 \sin \phi \]

Since for all AGMA gears \( a = \frac{d}{N} = \frac{2r_o}{N} \)

\[ z_G = -\sqrt{r_0^2 \left(1 + \frac{2}{N_0}\right)^2 - r_0^2 \cos^2 \phi} - r_0 \sin \phi \]
\[ z_G = r_0 \left[ -\sqrt{(1 + \frac{2}{N_0})^2 - \cos^2 \phi} - \sin \phi \right] \]

The pinion would have a similar segment of the overall length of action,

\[ z_p = r_p \left[ -\sqrt{(1 + \frac{2}{N_p})^2 - \cos^2 \phi} - \sin \phi \right] \]

The length of action for the top equation is then

\[ Z = z_G + z_p \]

Contact ratio approximates the average number of teeth in contact as the gears rotate.

Rule of thumb: \( 1.4 \leq m_p \leq 2.0 \)